1 Introduction

An initial configuration, a goal configuration, an airplane and obstacles are given. The path planning algorithm finds a path from the initial configuration to the goal one while avoiding obstacles. The path planning algorithm uses potential field method. That is, the algorithm greedily finds the next position without searching all the possible paths. The goal position pulls the plane with attractive force while the obstacles pushes the plane with repulsive force.

1.1 Configuration

Configuration for a plane is composed of 6 elements ($x$, $y$, $z$, $\alpha$, $\beta$, $\gamma$). $\alpha$ is a counterclockwise rotation about the z axis (yaw). $\beta$ is counterclockwise rotation about the y axis (pitch). $\gamma$ is a counterclockwise rotation about the x axis.

1.2 Control Points

The picked three control points are on the plane: one is at the head of plane; the other two points are at the end of wings (left and right). Given the 3D object of plane, the head point is approximately at $(0.7, 0, 0)$. The others are respectively at $(-0.7, 0.7, 0)$ and $(-0.7, -0.7, 0)$.

2 Forward Kinematics

These rotations are a reference [2]

2.1 yaw

$$R_z(\alpha) = \begin{pmatrix}
\cos \alpha & -\sin \alpha & 0 \\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{pmatrix}.$$
2.2 pitch

\[ R_\theta(\beta) = \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix}. \]

2.3 roll

\[ R_\theta(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{pmatrix}. \]

\[ Rotation = R_x(\alpha)R_y(\beta)R_z(\gamma) = \begin{pmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma \\ \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma \\ -\sin \alpha \cos \gamma & \cos \alpha \sin \gamma & \cos \alpha \cos \gamma \end{pmatrix}. \]

For easy computation, the origin of plane is translated into (-0.7, 0, 0). Thus, the control points are respectively at (1.4, 0, 0), (0, 0.7, 0) and (0, -0.7, 0). The Forward Kinematics for the control point in head of the plane is as follows.

\[ \theta_{head}(q) = \begin{pmatrix} x + 1.4 \cos \alpha \cos \beta \\ y + 1.4 \sin \alpha \cos \beta \\ z - 1.4 \sin \beta \end{pmatrix}. \]

The control point in the left wing of the plane is as follows

\[ \theta_{left}(q) = \begin{pmatrix} x + 0.7 \cos \alpha \sin \beta \sin \gamma - 0.7 \sin \alpha \cos \gamma \\ y + 0.7 \sin \alpha \sin \beta \sin \gamma + 0.7 \cos \alpha \cos \gamma \\ z + 0.7 \cos \beta \sin \gamma \end{pmatrix}. \]

The control point in the right wing of the plane is as follows

\[ \theta_{right}(q) = \begin{pmatrix} x - 0.7 \cos \alpha \sin \beta \sin \gamma + 0.7 \sin \alpha \cos \gamma \\ y - 0.7 \sin \alpha \sin \beta \sin \gamma - 0.7 \cos \alpha \cos \gamma \\ z - 0.7 \cos \beta \sin \gamma \end{pmatrix}. \]

3 Jacobian Matrix

The Jacobian Matrix for the control point in head of the plane is as follows.

\[ \theta_{head}(q) = \begin{pmatrix} 1 & 0 & 0 & -1.4 \sin \alpha \cos \beta & -1.4 \cos \alpha \sin \beta & 0 \\ 0 & 1 & 0 & 1.4 \cos \alpha \cos \beta & -1.4 \sin \alpha \cos \beta & 0 \\ 0 & 0 & 1 & 0 & -1.4 \cos \beta & 0 \end{pmatrix}. \]

The control point in the left wing of the plane is as follows

\[ \theta_{left}(q) = \begin{pmatrix} 1 & 0 & 0 & -0.7 \sin \alpha \sin \beta \sin \gamma - 0.7 \cos \alpha \cos \gamma & 0.7 \cos \alpha \cos \beta \sin \gamma & 0.7 \cos \alpha \sin \beta \cos \gamma + 0.7 \sin \alpha \sin \gamma \\ 0 & 1 & 0 & 0.7 \cos \alpha \sin \beta \sin \gamma - 0.7 \sin \alpha \cos \gamma & 0.7 \sin \alpha \sin \beta \cos \gamma + 0.7 \cos \alpha \sin \gamma \\ 0 & 0 & 1 & 0 & -0.7 \sin \beta \sin \gamma & 0.7 \cos \beta \cos \gamma \end{pmatrix}. \]

The control point in the right wing of the plane is as follows

\[ \theta_{right}(q) = \begin{pmatrix} 1 & 0 & 0 & 0.7 \sin \alpha \sin \beta \sin \gamma + 0.7 \cos \alpha \cos \gamma & -0.7 \cos \alpha \cos \beta \sin \gamma & -0.7 \cos \alpha \sin \beta \cos \gamma - 0.7 \sin \alpha \sin \gamma \\ 0 & 1 & 0 & -0.7 \cos \alpha \sin \beta \sin \gamma + 0.7 \sin \alpha \cos \gamma & -0.7 \sin \alpha \sin \beta \cos \gamma + 0.7 \cos \alpha \sin \gamma \\ 0 & 0 & 1 & 0 & 0.7 \sin \beta \sin \gamma & 0.7 \cos \beta \cos \gamma \end{pmatrix}. \]
4 Forces

4.1 Attractive Forces

The attractive force is calculated with the equation in 102 page of the textbook [1]. $d_{goal}^*$ is assigned with 1. $\zeta$ is assigned 0.1.

4.2 Repulsive Forces

The attractive force is calculated with the equation in 102 page of the textbook. $Q^*$ is assigned with 1. $\eta$ is assigned 0.05.

References
