

CS 440 - Introduction to Artificial Intelligence

Homework 8

Due Date: Friday, Dec 8th, 12:30pm

1. (a) Suppose you are running a learning experiment on a new algorithm. You have a set consisting of 50 examples of each of two classes. You plan to use leave-one-out cross validation (i.e. you run the algorithm 50 times, each time taking out a different item for the test set, while using the other 49 as training examples.)

As a baseline, you run your experimental setup on a simple *majority classifier* which given a set of training data always outputs the class that is the majority in the training set, regardless of input. You expect the majority classifier to score about 50% on leave-one-out cross validation, but to your surprise, it scores 0. Can you explain why?

- (b) Suppose we generate a training set from a decision tree T (say, by making a random choice at each node and following that path until we reach a decision node.) Is it true that the decision tree learning algorithm on this training set will converge to T as the training set size goes to infinity? Why or why not?
2. In this problem, you will show that the value iteration algorithm for solving a Markov Decision Problem $M = (S, A, T, \gamma, R)$ with an infinite horizon converges to the optimal value function in a finite number of steps.

- (a) An *operator* L over a space of functions $f : X \mapsto Y$, is a mapping from one function in this space to another. Let L be an operator that takes a value function V for M and returns another value function LV that is the result of applying one step of the value iteration procedure to V . Show that L is a *contraction mapping* i.e.

$$\| LV - LV' \| \leq \gamma \| V - V' \|$$

where $\| f \| = \max_{x \in \text{dom } f} |f(x)|$.

- (b) Now show that $\| V^* - LV \| \leq \gamma \| V^* - V \|$ where V^* is the optimal value function.
- (c) Argue that for any initial value function V^0 , there exists some k s.t. $V^k = V^*$, where $V^k = LV^{k-1}$ for $k \geq 1$. Note that this is stronger than the statement $\lim_{k \rightarrow \infty} V^k = V^*$.