UIUC, CS498, Section EA - Autumn 04 - Midterm Sample

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1 Short Questions (25%)

1. Determine whether each of the following are valid, satisfiable (but not valid), or unsatisfiable. No explanation necessary.
   (a) $(happy \Rightarrow productive) \wedge happy \wedge \neg productive$
   (b) $(happy \Rightarrow productive) \wedge happy \wedge unproductive$
   (c) $\neg p \wedge (\neg(h \Rightarrow p) \vee (\neg p \wedge h))) \vee (\neg h \wedge \neg p) | \neg(\neg p \wedge \neg h)$
   (d) $\neg(\neg r \vee t \vee \neg s) \vee \neg(r \Rightarrow s) \vee (r \Rightarrow t)$
   (e) $(\neg r \wedge (s \iff \neg(q \vee r))) \iff (\neg s \vee \neg(t \Leftarrow (r \wedge q)))$
2. Consider the following set of sentences we will refer to by Δ.

$$\begin{array}{l} logician(post) \\ logician(godel) \\ logician(turing) \\ logician(church) \\ \\ intelligent(post) \\ intelligent(godel) \\ intelligent(turing) \\ intelligent(church) \\ intelligent(frodo) \end{array}$$

   (a) Does $\Delta \models \forall x.(logician(x) \wedge intelligent(x))$? Answer Yes or No. If no give a model that demonstrates this fact.
   (b) Does $\Delta \models \forall x.(logician(x) \Rightarrow intelligent(x))$? Answer Yes or No. If no give a model that demonstrates this fact.

2 Inference (15%)

1. Convert this sentence to clausal form

$$\neg p \vee (q \Rightarrow \neg(r \iff q))$$

2. Using propositional linear resolution, show the following propositional sentence is unsatisfiable.

$$(p \vee q \vee \neg r) \wedge ((\neg r \vee q \vee p) \Rightarrow ((r \vee q) \wedge \neg q \wedge \neg p))$$

   To do this, convert this sentence to clausal form and derive the empty clause using resolution.

3 Relational Statements (15%)

1. Translate the following english sentences into relational logic. No explanation necessary.

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Object constant: 0
Relations:
   W(x): x is a whole number, i.e. 1, 2, 3, ...
   N(x): x is a natural number, i.e. 0, 1, 2, 3, ...
   Q(x): x is a rational number
   R(x): x is a real number
   x > y: x is greater than y
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   (a) "Every natural number greater than 0 is a whole number."

   (b) "Some real numbers are rational numbers, but some aren't."

   (c) "For every pair of natural numbers there is a rational number between them."

4 Propositional Logic (25%)

Lyndon's Interpolation Theorem for propositional logic is the following:
If $\alpha \models \beta$, and α, β propositional sentences, then there is a sentence γ that includes only literals that appear in both α, β such that $\alpha \models \gamma$ and $\gamma \models \beta$.

Prove Lyndon's Interpolation Theorem for propositional logic from Craig's interpolation theorem for propositional logic (hint: create new symbols).

5 First-Order Logic (20%)

Let Δ be an arbitrarily-sized but finite set of first-order sentences. Let ϕ be a first-order sentence. Prove $\Delta \models \phi \iff \Delta \cup \{\neg\phi\} \models \{\}$.