

UIUC, CS498, Section EA - Autumn 04 - Homework #2

Due Date: Tuesday, October 26, 2004, 12pm

October 11, 2004

1 First-Order Logic (30%)

1. A set of theories \mathcal{T} has the *finite-model property*, if whenever $T \in \mathcal{T}$ then T also has a finite model. Let \mathcal{T}_1 be the set of monadic first-order theories with vocabulary L , and without equality. Prove that \mathcal{T}_1 has the finite-model property.
2. A first-order clause C is a *prime implicate* of clausal theory T , if whenever clause D strictly subsumes C (i.e., C does not subsume D), then $T \not\models D$.
 - (a) Prove or disprove: Every clausal theory T is equivalent to the conjunction of its prime implicates.
 - (b) Prove that if a theory T is equivalent to the conjunction of its prime implicates, and it has finitely many prime implicates, then inference with T is decidable, i.e., there is a procedure for answering a query $T \models \varphi$ that always terminates.

2 Resolution in FOL (50%)

1. Consider an image provided by a camera with 100x100 pixels. We consider the 2-D objects in the image (you can assume that they are convex opaque polygons). These objects may obscure each other, but there cannot be a sequence of objects (polygons) P_1, \dots, P_n such that P_{i+1} obscures a part of P_i , for every i , and $P_1 = P_n$. Each pixel belongs to the boundary of an object or to the surface of an object. If adjacent pixels are both surface pixels, then they must belong to the same object. Two objects are always separated by at least one boundary (the one of the obscuring object (if one is obscuring part of the other) or both boundaries (if they touch)). The boundary of an object is always a single closed path (but parts of that path can be obscured). Assume also that the image we get has already gone through an edge detector, and so the image is in black/white (0/1) with boundaries colored white, and the rest in black.
 - (a) Represent this information in FOL.
 - (b) Convert the FOL sentences to clausal form.
 - (c) Download a FOL theorem prover (recommended: Vampire, SPASS, Otter, PTTP, in this order, but you can use whichever FOL theorem prover you choose). Prove from your theory that every path between the surfaces of two objects has to cross at least one boundary line.
 - (d) Represent 5 different polygons using your FOL language. Generate an image that includes those 5 polygons, with some obscuring parts of others. Use your chosen theorem prover to find three polygons in the image.

3 Strategies (20%)

1. Prove that unit resolution is complete for Horn clauses (you can do so in propositional logic).
2. Find examples that show that the shortest proof using Set-Of-Support resolution is shorter than shortest proofs in Linear resolution and vice versa.
3. Prove that partitioned reasoning with two partitions is complete for first-order logic (you can use the fact that resolution is complete for consequence finding).