If you miss more than 4 classes (you do not attend) during the semester, strongly consider dropping the class.

+ 2 late days.

Independence  \[ \text{Conditional Probability} \]

Def: \( X, Y \) independent if \( \Pr(X/Y) = \Pr(X) \).

Input: Joint probability \( \frac{\Pr(X, Y)}{\Pr(X)} \)

Marginal = \( \Pr(X_i = x_i) \)

Basic Concept: Conditional Independence

Structure: Bayesian Networks

\[ \frac{\Pr(X, Y)}{\Pr(X)} = \Pr(Y/X) \] Bayes Rule

\[ \Pr(X) \] Chain Rule

\[ \Pr(X, Y) = \Pr(X) \cdot \Pr(Y/X) \]

\[ \Pr(X_1, ..., X_n) = \Pr(X_1) \cdot \Pr(X_2/X_1) \cdot ... \cdot \Pr(X_n/X_1, ..., X_{n-1}) \] Product Rule
Independence

\[ P_r(X \mid Y) = P_r(X) \iff X, Y \text{ independent} \]

**Claim:** \( P_r(Y \mid X) = P_r(Y) \), given \( P_r(X \mid Y) = P_r(X) \).

**Pf:**

\[
P_r(Y \mid X) = \frac{P_r(X, Y)}{P_r(X)} = \frac{P_r(Y) \cdot P_r(X \mid Y)}{P_r(X)} = \frac{P_r(Y) \cdot P_r(X)}{P_r(X)} = P_r(Y)
\]

\( \text{Bayes Chain Rule} \)

\( X, Y \text{ indep.} \)

\[ \boxed{-2} \]

**Claim:** \( P_r(X, Y) = P_r(X) \cdot P_r(Y) \), if \( X, Y \text{ indep.} \).

**Pf:**

... work at home (in 1 min)

\[ \square \]

**Task:** If we classify 10 types of hair colors and have 68 people in CS440 + Staff, how many entries in the Joint Probability table?

\[ P_r(C_1 \ldots C_{68}) \]

1068

\[ \text{rows ?} \]

Example from before:

\[ \begin{array}{ccc}
\text{C}_1 & \text{C}_2 & \text{C}_3 \\
\text{C}_4 & \text{C}_5 & \text{C}_6 \\
\text{C}_{68} & & \\
\end{array} \]

We will be given these parameters

\[ P_r(B, E, A, R, C) = \]

\[ = P_r(E) \cdot P_r(B) \cdot P_r(A \mid B, E) \cdot P_r(R \mid E) \cdot P_r(C \mid A) \]
Using independence: \( \Pr(C_i \mid C_{68}) = \prod_{i=1}^{68} \Pr(C_i) \)

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( \Pr(C_1) )</th>
<th>( C_{68} )</th>
<th>( \Pr(C_{68}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( p_1^* )</td>
<td>1</td>
<td>( p_{68}^* )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>10</td>
<td>( p_{10}^* )</td>
<td>10</td>
<td>( p_{10} )</td>
</tr>
</tbody>
</table>

Questions: How many parameters \( (p_i^*) \) do I need to specify \( \Pr(C_1 \cdots C_{68}) \) ?

Answer: \( 68 \times 9 \)

\( p_0^* = 1 - \sum_{i=1}^{9} p_i^* \)

Definition: With a DAG \( G(V,E) \)

\[ V = \{ X_1, \ldots, X_n \} \]

\[ \Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} \Pr(X_i \mid Pa(X_i)) \]

Example from before:

We will be given these parameters:

\[ \Pr(B, E, A, R, C) = \]

\[ = \Pr(E) \cdot \Pr(B) \cdot \]

\[ \cdot \Pr(A \mid B, E) \cdot \]

\[ \cdot \Pr(R \mid E) \cdot \Pr(C \mid A) \]