1. Convert the following propositional formulae into Conjunctive Normal Form (CNF). For each formula, figure out the number of its models, state whether it is satisfiable and valid, satisfiable but not valid, valid but not satisfiable, or neither valid nor satisfiable.

   (a) \((x_1 \land \neg x_2 \land x_3) \lor (x_1 \land (x_3 \Rightarrow x_4))\)
   (b) \((x_1 \Rightarrow \neg(x_2 \land x_3)) \Leftrightarrow ((x_1 \Rightarrow \neg x_2) \land (x_1 \Rightarrow \neg x_3))\)

2. Prove the following assertions and statements:

   (a) \(\alpha \models \beta\) iff \(\neg \alpha \lor \beta\) is valid.
   (b) \(x \models \neg \neg x\)

3. Linear Algebra

   (a) Let \(A = \begin{pmatrix} 0 & 3 & 3 \\ 1 & 1 & 0 \\ -1 & 2 & 3 \end{pmatrix}\)

      i. Calculate the rank of \(A\), what does it mean to the following linear system?

         \[
         \begin{align*}
         3x_2 + 3x_3 &= 1 \\
         x_1 + x_2 &= 2 \\
         -x_1 + 2x_2 + 3x_3 &= 3
         \end{align*}
         \]

      ii. Give a matrix \(B\) s.t. \(AB = A + 2B\).

   (b) Assume vector \(\alpha\) is an eigenvector of matrix \(A\)

      \[
      A = \begin{pmatrix} a & 2 & -2 \\ 2 & b & -4 \\ -2 & -4 & 5 \end{pmatrix}, \quad \alpha = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}
      \]

      Give the values of \(a\) and \(b\).

4. Probability

   (a) Suppose the scores of Homework 1 can be approximately described by a Gaussian distribution, however the parameters, mean \(\mu\) and variance \(\sigma^2\), are unknown. TA randomly picked 10 copies and scored them, the scores are recorded below:

      \[78\quad 83\quad 85\quad 67\quad 95\quad 89\quad 87\quad 80\quad 81\quad 84\]

      i. Can you help TA to estimate the parameters? (Note: there might be multiple estimators, choose one based on your preference and give a simple defense for your choice.)
ii. Based on your previous estimation, what’s the probability a student has a grade greater than 90?

(b) According to WILL, 20% population of Champaign-Urbana area has taken H1N1 vaccines. Researches show that the infection rate of H1N1 is around 50%, and the effectiveness of the vaccine is around 96%.

i. If someone is from C-U area, what is the probability he got infected if exposed to H1N1 virus?

ii. If one student is diagnosed being infected by H1N1 at McKinley, what’s the probability this student has taken vaccine?